LETTERS TO THE EDITOR

To the editor:

I recently found that there was an error in the paper, entitled Characterization and Analysis of Continuous Recycle Systems," by Uzi Mann, Michael Bubinovitch and Edwin J. Crosby, [*AIChE J.* **25,** 873 (1979)]. The equation (43) represented the residence time distribution of the total system of Case D may be incorrect, though the equation (41), (42), which resulted in the equation (43) and represented the probability density of flow regime (1), (2) respectively were right.

$$h_1(x) = \frac{1}{\Gamma(\alpha_1)}\cdot\frac{1}{\beta_1}\left(\frac{x-\tau_1}{\beta_1}\right)^{\alpha_1-1} e^{-(x-\tau_1)/\beta_1}\,(x \geqslant \tau_1) \qquad (41)$$

$$h_2(y) = \frac{1}{\tau(\alpha_2)}\cdot\frac{1}{\beta_2}\left|\frac{y-\tau_2}{\beta_2}\right)^{\alpha_2-1-(y-\tau_2)/\beta_2}\,(y \geqslant \tau_2) \qquad (42)$$

$$f(t) = (1-p)\cdot\frac{1}{\Gamma(\alpha_1)}\cdot\frac{1}{\beta_1}\left(\frac{t-\tau_1}{\beta_1}\right)^{\alpha_1-1} e^{-(t-\tau_1)/\beta_1} + (1-p)\sum_{n=2}^{\infty} p^{n-1}\frac{1}{\Gamma(n\alpha_1)}\cdot\frac{1}{\Gamma((n-1)\alpha_2)}\cdot\frac{1}{\beta_1}\cdot\frac{1}{\beta_2}$$
$$\int_{t-h\tau_1-(n-1)\tau_2}^{t}\left(\frac{x-n\tau_1}{\beta}\right)^{n\alpha_1} e^{-(x-\tau_1)/\beta_1}\left(\frac{t-n\tau_1-(n-1)\tau_2-x}{\beta_2}\right)^{(n-1)\alpha_1-1}\cdot e^{-(t-n\tau_1-(n-1)\tau_2-x)\beta_2}\,dx \qquad (43)$$

Using the theorem of convolution, the convolution of $h_1(x)$ and $h_2(y)$ should be the following:

$$f(t) = (1-p)\frac{1}{\Gamma(\alpha_1)}\cdot\frac{1}{\beta_1}\left(\frac{t-\tau_1}{\beta_1}\right)^{\alpha_1-1} e^{-(t-\tau_1)/\beta_1} + (1-p)\sum_{n=2}^{\mathrm{int.}((t+\tau_2)/\tau_2)} p^{n-1}\cdot\frac{1}{\Gamma(n\alpha_1)}$$
$$\cdot\frac{1}{\Gamma((n-1)\alpha_2)}\cdot\frac{1}{\beta_1}\cdot\frac{1}{\beta_2}\int_{n\tau_1}^{t-(n-1)\tau_2}\left(\frac{x-n\tau_1}{\beta_1}\right)^{n\alpha_1} e^{-(x-n\tau_1)/\beta_1}\cdot\left(\frac{t-(n-1)\tau_2-x}{\beta_2}\right)^{(n-1)\alpha_2-1}\cdot e^{-(t-(n-1)\tau_2-x)/\beta_2}\,dx$$

There were some printing errors in the equation (43), also. However, some errors were not due to the carelessness in printing.

As $x - n\tau$, and $t - (n-1)^*\tau_2 - x$ must be greater than or equal to 0, i.e., $x \geqslant n\,\tau_1$ and $x \leq t - (n-1)\tau_2$, the upper and lower limits of the integral should be $n\,\tau_1$ and $t - (n - 1)^*\tau_2$, respectively.

In all the four cases mentioned in the paper, the value of n, was not an infinity. In fact, it was restricted by certain conditions. For example, in the equation (43), the upper limit term $t - (n-1)\cdot\tau_2$ must be greater than 0, i.e., the value of n should be the integer of the ratio of $(t + \tau_2)/\tau_2$, and therefore, it can be determined by the variable t. This is an important point, especially in the calculation of f(t), so it should be mentioned in the paper.

With best regards to you and to the authors and waiting for the author's opinion at their earliest convenience.

Zuang-Cong Lu
Graduate Student in the Research Institute
of Chem. Eng. Shanghai, China

To the editor: On "Pressure Drop for Cocurrent Downflow of Gas-Solids Suspensions"

Kim and Seader (1983), generalizing the relation between the two-phase friction factor f_{tp} and dimensionless parameter α by applying it to a packed bed, have obtained the following correlation

$$f_{tp} = 0.85\alpha$$

This has been done by recorrelating of pressure drop values in a packed bed as given by Ergun's equation (Ergun, 1952). Later on, they have suggested that the intermediate regions, i.e., moving bed, fluidized bed, etc., may also be correlated in terms of f_{tp} and α, but they considered it as a proposition for a further investigation. It is interesting indeed to show if that suggestion is right.

Ergun's equation (1952):

$$\frac{\Delta P_f}{L} = \frac{150\,E_p^2}{(1-E_p)^3}\frac{\mu_g U_{sg}}{d_p^2} + 1.75\frac{E_p}{(1-E_p)^3}\frac{\rho_g U_{sg}^2}{d_p} \qquad (2)$$

expressed as a function of the friction factor takes the following form

$$f_{tp} = \frac{E_p}{(1-E_p)^3}\frac{D}{2\,d_p}f_E, \qquad (3)$$

where

$$f_E = \frac{150\,E_p}{\frac{U_{sg}d_p\rho g}{\mu_g}} + 1.75 \qquad (4)$$

But the Ergun's equation for a packed bed at the point of incipient fluidization or any further state of fluidization can be rearranged into the form

$$f_E\frac{Re_p^2}{Ar} = (1-E_p)^3 \qquad (5)$$

This relation is obtained under assumption

$$\Delta P_f = g\,E_p\,L(\rho_p - \rho_f), \qquad (6)$$

which is generally accepted for a fluidized bed. On the other hand the expansion of a particular fluidized bed can be described by the correlation (Kmiec, 1982)

$$\frac{3}{4}(C_{D,s})_{U_{sg}}(1-E_p)^{-4.78}\frac{Re_p^2}{Ar} = 1, \qquad (7)$$

which corresponds exactly to the relation of Wen and Yu (1966). In the above formula, $(C_{D,s})U_{sg}$ is the drag coefficient of particles in an infinite fluid based on the superficial velocity of fluid (U_{sg}). This equation can be rewritten

$$\frac{3}{4}\frac{C_D}{(1-E_p)^2}\frac{Re_p^2}{Ar} = 1, \qquad (8)$$

where C_D is the drag coefficient based on the actual intersticial velocity. Combining equations (5) and (8) the relation between the coefficient f_E and the drag coefficient C_D is obtained

$$f_E = \frac{3}{4} C_D (1 - E_p) \qquad (9)$$

Introducing the above equation into equation (3) the following correlation between f_{tp} and α for a fluidized bed is obtained

$$f_{tp} = \left[\frac{3}{8(1-E_p)}\right]\alpha \qquad (10)$$

It is seen that in the case of a fluidized bed there is also a factor being a recipro-cal of bed porosity. Assuming that for a packed bed at the point of incipient fluidization the porosity is equal to 0.441 we obtain the same value of coefficient as in equation (1).

Andrzej Kmiec
Institut für Mechanische Verfahrenstechnik
Technische Universität Clausthal
D3392 Clausthal-Zellerfeld
West Germany

ACKNOWLEDGMENT

The author is grateful to the Alexander von Humboldt-Stiftung of West Germany for a fellowship.

NOTATION

- Ar = Archimedes number, $g\, d_p^3 (\rho_p - \rho_f)\rho_f/\mu_f^2$
- C_D = drag coefficient in a suspension
- $C_{D,s}$ = drag coefficient of a single particle in an infinite medium
- d_p = particle diameter
- D = tube diameter
- E_p = particle holdup
- f_{tp} = two-phase friction factor, $f_{tp} = \dfrac{\Delta P_f D}{2\rho_g U_{sg}^2 L}$
- g = acceleration due to gravity
- L = bed length
- Re_p = Reynolds number, $(U_{sg} d_p \rho_g/\mu_g)$
- ΔP_f = pressure drop
- U_{sg} = superficial fluid velocity

Greek Letters

- α = dimensionless parameter, $(C_D E_p D/[(1-E_p)d_p])$
- μ_g = gas viscosity
- ρ_g = gas density
- ρ_p = particle density

LITERATURE CITED

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Kmiec, A., "Equilibrium of Forces in a Fluidized Bed-Experimental Verification', Chem. Eng. J. (Lausanne), 23, 133 (1982).
Wen, C. Y., and Y. H. Yu, "Mechanics of Fluidization", Chem. Eng. Prog., Symp., 62, 100 (1966).

ERRATA

In "Characterization and Analysis of Continuous Recycle Systems: Part I. Single Unit" by Mann, et al. [*AIChE J.* **25**, 873 (1979)], a few equations were printed with errors. Some of the errors were pointed out by Zuang-Cong Lu of the Research Institute of Chemical Engineering in Shanghai, China. The corrections are as follows:

1. The term q(j) instead of q(n) should appear in the summation of Eq. 15.
2. The last line of Eq. 23 should be deleted.
3. It should be pointed out that the density functions of the RTD, f(t), for all the four cases discussed (Eqs. 26, 32, 38 and 43) are defined for $t - n\tau_1 - (n-1)\tau_2 \geq 0$ and are equal to zero otherwise.
4. The last line of Eq. 38 should be

$$\left(\frac{t - n\tau_1 - (n-1)\tau_2}{\beta_2}\right)^{(n-1)\alpha_2 - 1} \cdot e^{-(t-n\tau_1-(n-1)\tau_2)/\beta_2}$$

5. In Eq. 43, the limits of the integral and the upper limit of the summation are incorrect. Also, the equation begins on page 879 and ends on page 878. The complete equation is as follows:

$$f(t) = (1-p)\frac{1}{\Gamma(\alpha_1)}\frac{1}{\beta_1}\left(\frac{t-\tau_1}{\beta_1}\right)^{\alpha_1-1} \cdot e^{-(t-\tau_1)/\beta_1}$$
$$+ (1-p)\sum_{n=2}^{(t+\tau_2)/(\tau_2+\tau_2)} p^{n-1}\frac{1}{\Gamma(n\alpha_1)}\frac{1}{\Gamma((n-1)\alpha_2)}\frac{1}{\beta_1}\frac{1}{\beta_2} \cdot$$
$$\cdot \int_{n\tau_1}^{t-(n-1)\tau_2}\left(\frac{x-n\tau_1}{\beta_1}\right)^{n\alpha_1-1}$$
$$\times e^{-(x-n\tau_1)/\beta_1}\left(\frac{t-(n-1)\tau_2-x}{\beta_2}\right)^{(n-1)\alpha_2-1}$$
$$\cdot e^{-(t-(n-1)\tau_2-x)/\beta_2}\,dx$$

6. Eq. 62 should be

$$E[N,T] = -\frac{d}{ds}\frac{d}{dz}\hat{G}(z,s)\Big|_{\substack{s=0\\ z=1}}$$

Uzi Mann and Michael Rubinovitch
Department of Chemical Engineering
Texas Tech University
Lubbock, Texas 79409